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Damage assessment in beams using vibration characteristics

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Keywords: structural health monitoring (SHM); damage detection; bridges; vibration; modal analysis

ABSTRACT: Vibration-based damage detection methods use measured changes in vibration characteristics to evaluate changes in physical properties that may indicate structural damage or degradation. The basic idea is that modal parameters (notably frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Changes in the physical properties will therefore cause changes in the modal properties. Any reduction in structural stiffness and increase in damping in the structure may indicate structural damage or degradation. This paper develops a comprehensive procedure based on these principles, to non-destructively assess damage in beam type of bridges, using finite element techniques supported by experiments. Using the results from modal analysis of calibrated finite element models, algorithms based on flexibility and strain energy changes before and after damage are obtained and used as the indices for the assessment of the structural status. The effectiveness of the proposed procedure is demonstrated via numerical examples.

1 INTRODUCTION

Bridges are an important and integral part of modern transportation systems and play a vital role in the lives of a community. Bridge failure or poor performance will disrupt the transportation system and may also result in loss of lives and property. It is therefore very important to ensure that bridges perform safely and efficiently at all time by monitoring their structural integrity and undertaking appropriate remedial measures. Many of the bridges in Brisbane (and in Queensland) had been designed and built several decades ago. Today, these structures are subjected to heavier and faster moving loads, compared to their original design loads. As an example, significant levels of vibration had been monitored in some spans of the Storey Bridge in Brisbane, which was built some 60 years ago (Thambiratnam, 1995). Others, such as the Victoria Bridge in Brisbane have been subjected to modified use to accommodate bus lanes on one side and this can lead to torsional vibration which may not have been accounted for. These changes in loading patterns, together with normal deterioration with age, can bring about localised failure and if this goes undetected the failure can extend and cause partial or even total collapse of the structure. At the time of writing this paper, there is an interest in the Brisbane City Council and the Queensland Main Roads Department (the owners of

the bridges) to monitor some of the bridges in order to evaluate their performance and carry out retrofitting.

In recent times, structural health monitoring (SHM) based on variations in vibration characteristics have emerged as an efficient technique. Health monitoring techniques based on processing vibration measurements basically handle two types of characteristics: the structural parameters (mass, stiffness, flexibility, damping) and the modal parameters (modal frequencies, mode shapes and damping ratios). As the dynamic characteristics of a structure, namely natural frequencies and mode shapes, are known to be functions of its stiffness and mass distribution, variations in modal frequencies and mode shapes can be an effective indication of bridge deterioration. Deterioration of a structure results in a reduction of its stiffness which causes the change in its dynamics characteristics. Thus, monitoring the change in these dynamic characteristics enables us to infer structural deterioration.

The objective of this paper is to evaluate the effectiveness of using the changes of natural frequencies, modal flexibility matrix and modal strain energy to non-destructively evaluate single and multiple damages in beam structures.

2 THEORY

2.1 Modal flexibility matrix

The modal flexibility includes the influence of both the mode shapes and the natural frequencies. It is defined as the accumulation of the contributions from all available mode shapes and corresponding natural frequencies. The modal flexibility matrix associated with the referenced degrees of freedom can be established from the following equation. (Huth et al. 2005)

$$[F] = [\phi][1/\omega^2][\phi]^T \quad (1)$$

$$\Delta[F] = [F^d] - [F^h] \quad (2)$$

where $[F]$ is the modal flexibility matrix; $[\phi]$ the mass normalized modal vectors; and $[1/\omega^2]$ a diagonal matrix containing the reciprocal of the square of natural frequencies in ascending order; the index 'h' refers to the healthy and the index 'd' to the damaged state. Theoretically, structural deterioration reduces stiffness and increases flexibility. Increase in structural flexibility can therefore serve as a good indicator of the degree of structural deterioration.

2.2 Modal strain energy based damage index

Cornwell et al. (1999) proposed this approach based on changes in modal strain energy of a damaged and an undamaged structure. The localisation of damage is based on the decrease in modal strain energy between two structural degrees of freedom as defined by the curvature of the mode shapes. Information required in the identification are the measured mode shapes and elemental stiffness matrices only without knowledge of the complete stiffness and mass matrices of the structure. The algorithm used to calculate the damage index β_{ji} for the j th element and i th mode of the beam is given below.

$$\beta_{ji} = \frac{k_j}{k_j^*} = \frac{\left(\int_j [\phi_i^{**}(x)]^2 dx + \int_0^L [\phi_i^{**}(x)]^2 dx \right) \int_0^L [\phi_i''(x)]^2 dx}{\left(\int_j [\phi_i''(x)]^2 dx + \int_0^L [\phi_i''(x)]^2 dx \right) \int_0^L [\phi_i^{**}(x)]^2 dx}$$

or rewritten as

$$\beta_{ji} = \frac{k_j}{k_j^*} = \frac{[(\phi_{ji}^{**})^2 + \sum (\phi_{ji}^{**})^2][\sum (\phi_{ji}'')^2]}{[(\phi_{ji}'')^2 + \sum (\phi_{ji}'')^2][\sum (\phi_{ji}^{**})^2]} \quad (3) \& (4)$$

To account for all available modes, a single indicator for each location along the beam is given by

$$\beta_j = \frac{\sum_{i=1}^{NM} Num_{ji}}{\sum_{i=1}^{NM} Denom_{ji}} \quad (5)$$

where Num_{ji} = numerator of β_{ji} and $Denom_{ji}$ = denominator of β_{ji} in Eq. (4)

3 METHOD

Finite element techniques will be used to carry out modal analysis of the beam structures. The finite element analysis (FEA) results are used to calculate the modal flexibility matrix and the modal strain energy based damage index and thereby assess the damage in the beam structures. The finite element model of the simply supported beam is first calibrated against the results from experimental testing. Further FEA is performed to determine the modal parameters of 2-span and 3-span continuous beams before and after damage. To simulate damage, the beams are cut to cause flaws on the tension face of the beam. Nine damage cases are investigated in this study. The details of modal testing and FE modelling are described below.

3.1 Experimental modal testing

Modal testing is conducted on the simply supported steel beam. The beam is tested in its undamaged condition to extract the pre-damage modal parameters. The test beam is excited by an impact hammer and the dynamic responses are measured by an accelerometer fixed at the mid-span of the beam. A software known as, SignalCalc ACE Dynamic Signal Analyzer is used to extract modal parameters. Then the test beam is cut to cause flaw at mid-span and the testing is repeated to extract the post-damage modal parameters.

3.2 Finite element modelling

A finite element model of the simply supported beam, tested previously, having a span length of 2.8m is generated using SAP2000 as shown in Fig. 1. Plane elements are used to model the beam. The details of the beam are given in Table 1. The flexural rigidity EI is assumed constant over the beam span. Modal analysis is performed to obtain the natural frequencies and the associated mode shapes of the beam. In order to validate and calibrate the FEM model, the two lowest natural frequencies

obtained from the laboratory experiments are compared to those from computer simulation. The experimental and finite element results showed good agreement as seen in Table 2. This calibrated baseline model is then used to evaluate the health of single and multiple damages on the beams by using the FEA results. Further finite element analysis (FEA) is performed to extract the modal parameters of 2-span and 3-span continuous beams. To simulate damage, the beam is cut to cause flaws on the tension face of the beam in FE modelling. Nine damages cases are investigated as shown in Fig. 3-11, with two different sizes of flaws as given in Table 3. The details of flaw size “A” simulated in FEM are shown in Fig. 2.

Table 1. Geometric and material properties of beam

Input parameters	Value
Element type	2D
Geometry type	Plane stress
Material	Isotropic
Width	40 mm
Depth	20 mm
Span	2.8 m
Boundary condition	Simply support
Poisson's ratio	0.3
Mass density	7850 kg/m ³
Modulus of elasticity	200 GPa

Table 2. Natural frequencies of simply supported beam:
Experimental testing vs FEM

State	f_i	Experiment (Hz)	FEM (Hz)	% Diff.
Undamaged	f_1	5.94	5.84	1.7
	f_2	24.38	23.33	4.3
Damaged at mid-span	f_1^*	5.63	5.65	0.4
	f_2^*	23.13	23.33	0.9

Note: f_i means i-th mode of frequency

Table 3. Dimension of flaws

Size	Length (mm)	Depth (mm)	Width (mm)
A	10	5	40
B	20	5	40



Fig. 1. Finite element model for single span beam

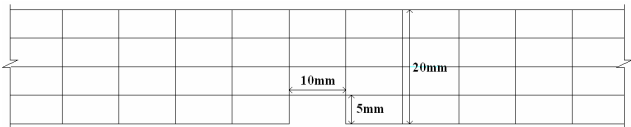


Fig. 2. Flaw size A simulated in FEM

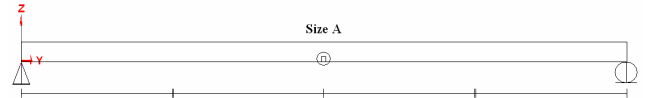


Fig. 3. Damage Case-1 for single span beam

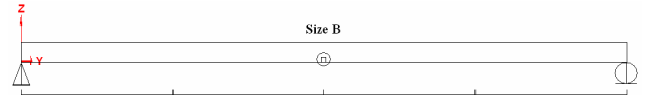


Fig. 4. Damage Case-2 for single span beam

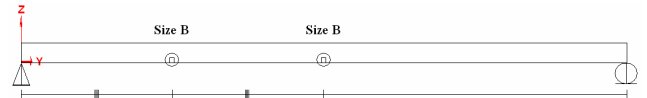


Fig. 5. Damage Case-3 for single span beam

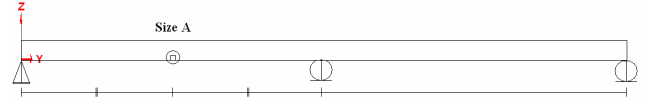


Fig. 6. Damage Case-4 for 2-span beam

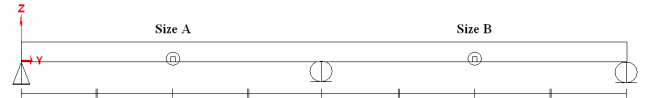


Fig. 7. Damage Case-5 for 2-span beam

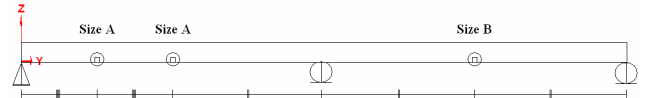


Fig. 8. Damage Case-6 for 2-span beam

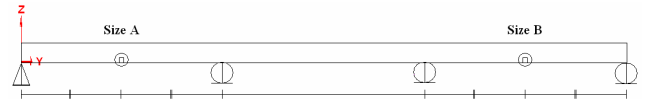


Fig. 9. Damage Case-7 for 3-span beam

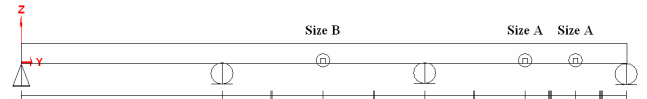


Fig. 10. Damage Case-8 for 3-span beam

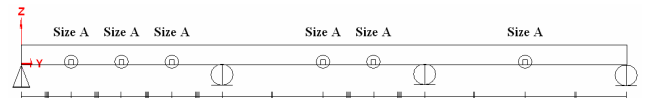


Fig. 11. Damage Case-9 for 3-span beam

Note: The span length is 2.8m for all beams

4 RESULTS AND DISCUSSIONS

4.1 Natural frequencies

The natural frequencies of the first five modes of the beams before and after damage in nine scenarios obtained from FEA result are shown in Table 4 & 5 respectively. Changes in the natural frequencies of beams between the damaged and undamaged conditions are shown in Table 6. As mentioned earlier the presence of damage causes a decrease in the natural frequencies.

Table 4. Natural frequencies of vibration from FEM
(Undamaged beams)

No. of span	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)	f_4 (Hz)	f_5 (Hz)
1	5.84	23.33	52.45	93.10	145.16
2	5.84	9.12	23.33	29.52	52.45
3	5.84	7.48	10.92	23.33	26.58

Table 5. Natural frequencies of vibration from FEM
(Damaged beams)

Damage Case	f_1	f_2	f_3	f_4	f_5
1	5.80	23.33	52.08	93.10	144.13
2	5.77	23.33	51.90	93.10	143.65
3	5.74	23.08	51.62	93.10	142.95
4	5.82	9.10	23.33	29.51	52.26
5	5.78	9.07	23.33	29.49	51.99
6	5.77	9.05	23.25	29.39	51.90
7	5.80	7.43	10.90	23.33	26.57
8	5.79	7.45	10.86	23.28	26.48
9	5.77	7.42	10.86	23.16	26.42

Table 6. Changes of natural frequencies (%) from FEM

Damage Case	f_1	f_2	f_3	f_4	f_5
1	0.68	0.00	0.71	0.00	0.71
2	1.20	0.00	1.05	0.00	1.04
3	1.71	1.07	1.58	0.00	1.52
4	0.34	0.22	0.00	0.03	0.36
5	1.03	0.55	0.00	0.10	0.88
6	1.20	0.77	0.34	0.44	1.05
7	0.68	0.67	0.18	0.00	0.04
8	0.86	0.40	0.55	0.21	0.38
9	1.20	0.80	0.55	0.73	0.60

4.2 Modal flexibility matrix

From FEA results, the first five modes of natural frequencies and associated mode shapes are used to calculate the change of modal flexibility matrix by using Eq. (1) & (2). The plot of percentage change of flexibility along the beam for damage cases 1-9 are shown in Fig. 12-17. The peak values indicate the location of damage in beams. From the results of the analysis, the damage localization algorithm based on the modal flexibility method is able to correctly detect and locate the damage of the beam structures. This confirms that the change of flexibility matrix is sufficiently sensitive to the damages on the beam structures and its increase in magnitude is a good indication for structural deterioration.

4.3 Strain energy based damage index

From FEA results, the first five mode shapes and its corresponding mode shape curvatures are used to calculate the modal strain energy based damage indices by using Eq. (4) & (5). The plot of damage in-

dice along the beam for damage cases 1-9 are shown in Fig. 18-23. Spikes with the magnitudes greater than 1 indicate the location of damage in beams. From the results of the analysis, the damage localization algorithm based on the modal strain energy method is able to correctly detect and locate the damage of the beam structures in most damage cases, except case 5 & 7 as they do not give any indication of the location of damage. Table 7 summarizes the performance of the damage localization algorithms in locating the damage on beams.

Table 7. Performance of damage localization analysis

Damage Case	Flexibility method	Strain energy method
1	✓	✓
2	✓	✓
3	✓	✓
4	✓	✓
5	✓	×
6	✓	✓
7	✓	×
8	✓	✓
9	✓	✓

Considering Fig. 12 & 13, it is clearly evident that as the severity of the single damage at mid-span increases, the corresponding percentage change in flexibility matrix also increases. Fig. 18 & 19 show the same feature with respect to the damage index. Research will continue to develop a procedure incorporating changes in natural frequency, modal flexibility matrix & damage index to accurately assess damage in beam structure.

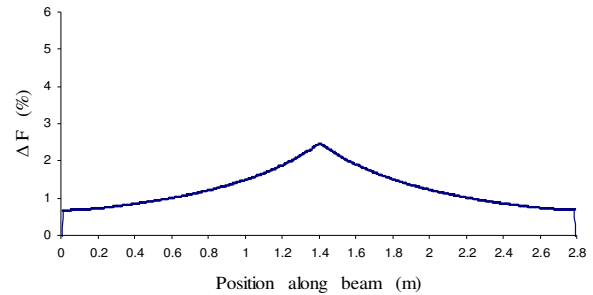


Fig. 12. Change of flexibility for single span beam (Case 1)

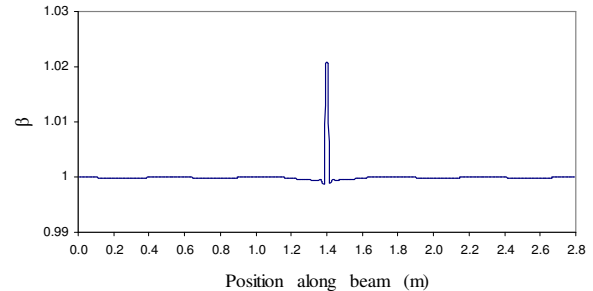


Fig. 18. Damage index for single span beam (Case 1)

Note: Figures for 3-span beams are not shown for want of space.

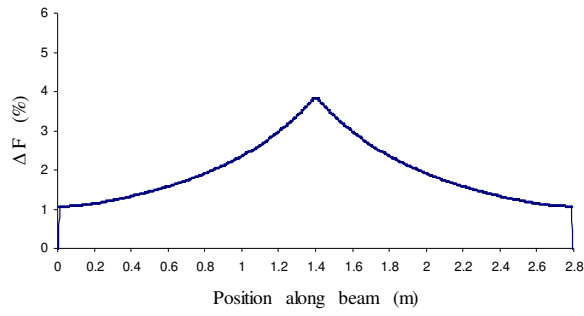


Fig. 13. Change of flexibility for single span beam (Case 2)

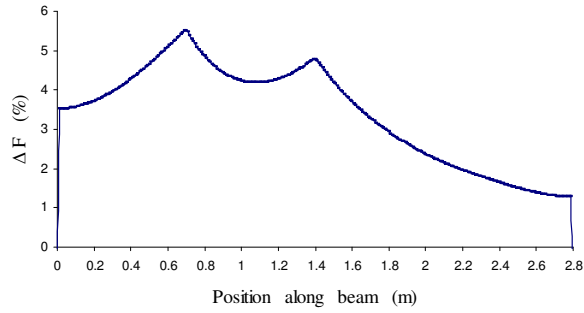


Fig. 14. Change of flexibility for single span beam (Case 3)

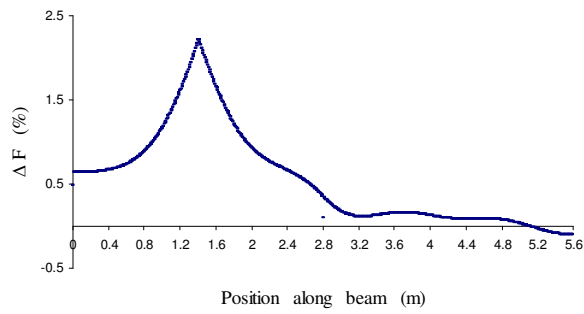


Fig. 15. Change of flexibility for 2-span beam (Case 4)

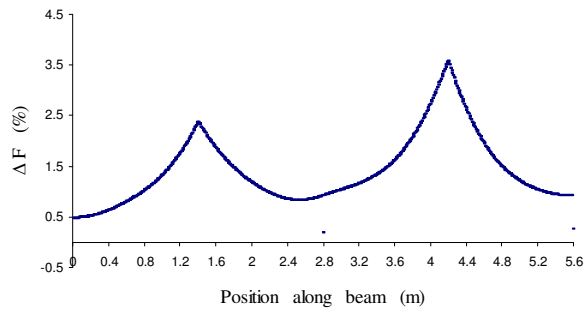


Fig. 16. Change of flexibility for 2-span beam (Case 5)

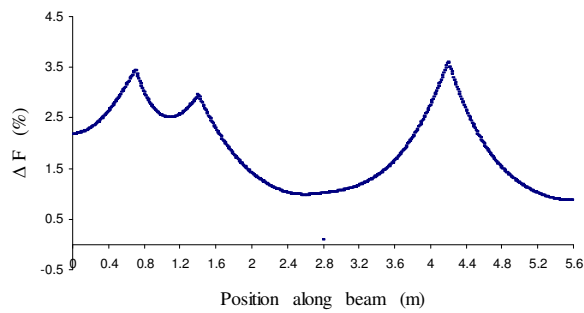


Fig. 17. Change of flexibility for 2-span beam (Case 6)

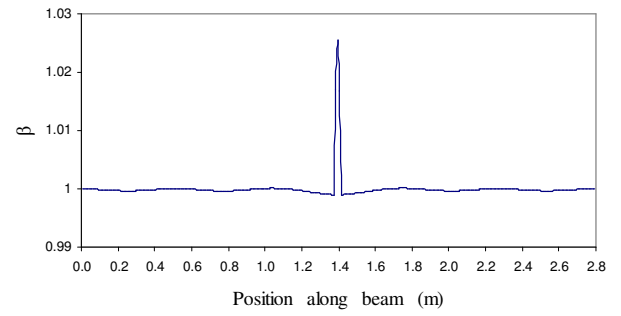


Fig. 19. Damage index for single span beam (Case 2)

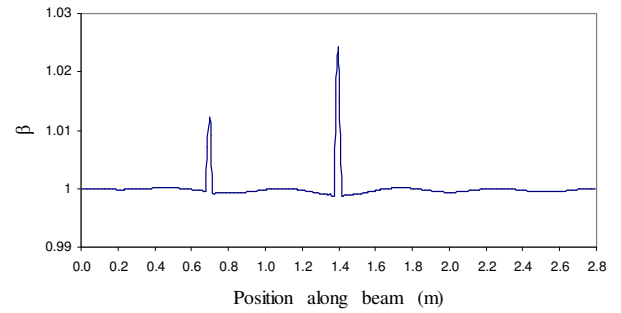


Fig. 20. Damage index for single span beam (Case 3)

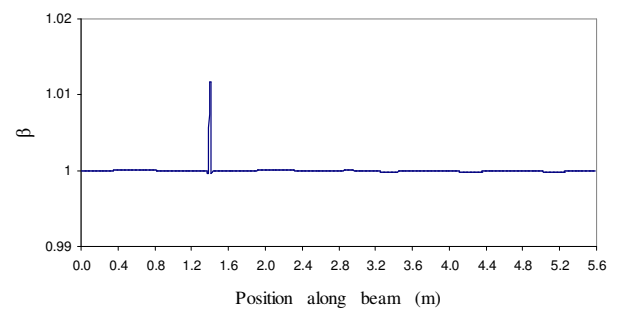


Fig. 21. Damage index for 2-span beam (Case 4)

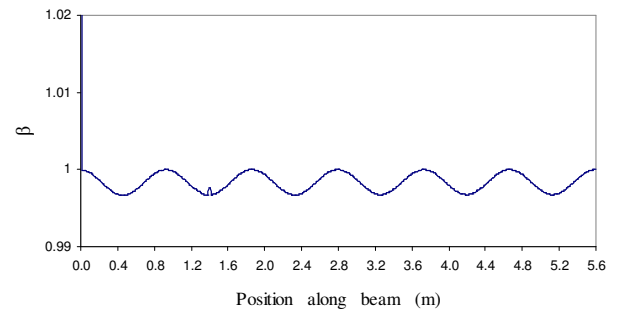


Fig. 22. Damage index for 2-span beam (Case 5)

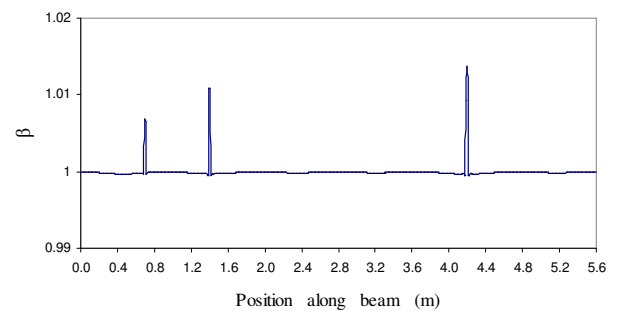


Fig. 23. Damage index for 2-span beam (Case 6)

5 CONCLUSIONS

In this paper, two damage localization algorithms (1) Modal flexibility matrix and (2) Modal strain energy based damage index are evaluated from the modal parameters to assess the state of health in beam structures. Changes in modal flexibility matrix and modal strain energy between the undamaged and damaged structure provide a basis for identification of localized damage. By applying the modal flexibility method to the beams, it is observed that the single or multiple damages can be confidently located with no localization error. Damage indices using strain energy changes successfully locate the single and multiple damages in most damage cases, except case 5 & 7. The implementation of second approach is more time consuming than the first approach for damage localization. It is concluded that the modal flexibility method appears to be more sensitive, precise and convenient to determine than the modal strain energy method for damage assessment of two dimensional beam structures. Changes in natural frequencies can be used to detect the presence of a state of damage, since this can be done from a single point measurement. Once the presence of damage is detected, modal flexibility method or modal strain energy method can be used to locate the damage. Future work will focus on a similar study for various types of damage on curved beams, truss and arch type bridges and also quantity damages in terms of variation of dynamic properties, i.e. the relationship between the change of modal flexibility matrix or modal strain energy with the size of flaw. The results will find application in assessment of the health of bridge structures.

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